The Essex Echo

Malcolm Omar Hawksford

Audiophiles are exited. A special event has occurred that promises to undermine their very foundation and transcend "the event sociological": a minority group now cite conductor and interconnect performance as a limiting factor within an audio system. The masses, however, are still content to congregate with their like-minded friends and make jokes in public about the vision of the converted, content to watch their distortion factor meters confidently null at the termination of any old piece of wire. Believing in Ohm's law, they feel strong in their brotherhood.

But the revolution moves forward. . .

This article examines propagation in cables from the fundamental principles of modern electromagnetic theory. The aim is to attempt to identify mechanisms that form a rational basis for a more objective understanding of claimed sonic anomalies in interconnects. Especially as I keep hearing persistent rumours about the virtues of single-strand, thin wires (John, is it OK to mention thin, single strand in HFN/RR yet? . . .)

Inevitably, the path towards an objective understanding depends upon both the correctness and completeness of the model selected. We shall establish, therefore, a theoretic stance initially and commence with the work of Maxwell (even though he could not avail himself of a distortion factor meter.) The equations of Maxwell concisely describe the foundation and principles of electromagnetism; they are central to a proper modelling of all electromagnetic systems. The equation set is presented below in standard differential form, where further discussion and background can be sought from a wide range of texts[1, 2, 3].

Maxwell's Equations:

Faraday's Law

$$\text{curl}\ \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

Ampere's Law

$$\text{curl}\ \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$$

Gauss's Theorem

$$\text{div}\ \overline{D} = \rho$$

No magnetic monopoles

$$\text{div}\ \overline{B} = 0$$

The constituent relationships that define electrical and magnetic material properties are,

$$bD = \varepsilon_0\varepsilon_r bE \qquad bB = \mu_0\mu_r bH$$

$bJ = \sigma\, bE$ (if σ is constant then this equation represents Ohm's law)

However, it is common to write $\varepsilon = \varepsilon_0\varepsilon_r$ and $\mu = \mu_0\mu_r$ thus $bD = \varepsilon bE$ and $bB = \mu bH$
where,

bE, electric field strength, (volt/m)

bB, magnetic flux density, (tesla)
bD, electric flux density, (coulomb/m^2)
ρ, charge density, (coulomb/m^3)
σ, conductivity, $(\text{ohm-m})^{-1}$ (conductivity is the reciprocal of resistivity)
bH, magnetic intensity, (ampere-(turn)-m)
bJ, current density, (ampere/m^2)
ε_0, permittivity of free space, (farad/m)
ε_r, relative permittivity
μ_0, permeability of free space, (henry/m)
μ_r, relative permeability
t, time, (second)

(The bar over some parameters indicate vector or directed quantities.)

It is relatively straightforward to show that the Maxwellian equation set is able to support a wave equation that governs the propagation of the electric and magnetic fields in both space and time. However, for those more interested in the sociological behavioural patterns of the ethnic minority of audiophiles, allow me a moment to describe the circumstances in which this article is being prepared.

The date is April 1st 1985 (honest). I have freed myself of the conservative British climate (political, weather and *audio*) and undertaken a transposition to the red-walled town of Marrakech in Morocco. The sky is clear and blue, the sun warm, yet the snow lies dormant on the Atlas mountains. The sound of a distant Arabic chant of the Koran sets a background, while the birds sing, watching the blossom develop on a multitude of orange trees, awaiting the fresh living fruit that matures in hours of endless sunshine, ready for my breakfast! There may yet not be a length of large crystal copper within a thousand mile radius. I shall check out the market place this afternoon, disguised with black beard and djelaba . . . ! That's strange. the snake charmer's snake has Snaic written on its side . . . ?

Let formal study commence. The wave equation describing a propagating electric field bE in a general lossy medium of conductivity σ, permittivity ε and permeability μ is derived as follows:

Operating by curl on the Faraday equation,

$$\mathrm{curl}(\mathrm{curl}\,\bar{E}) = -\,\mathrm{curl}\left(\frac{\partial \bar{B}}{\partial t}\right) = -\,\mu \frac{\partial}{\partial t}\,\mathrm{curl}\,\bar{H}$$

Substitute for curl bH from Ampere's law,

$$\mathrm{curl}(\mathrm{curl}\,\bar{E}) = -\,\mu \frac{\partial \bar{J}}{\partial t} - \mu \frac{\partial^2 \bar{D}}{\partial t^2}$$

Substitute also the Ohm's law relationship, bJ = σ bE and the vector identity,

$$\mathrm{curl}(\mathrm{curl}\,\bar{E}) = \mathrm{grad}(\mathrm{div}\,\bar{E}) - {}^2\bar{E}$$

where, assuming a charge-free region, div bD = 0, (i.e. div bE = 0), the generalised wave equation follows directly,

$$^2\bar{E} = \mu\sigma\frac{\partial \bar{E}}{\partial t} + \mu\varepsilon\frac{\partial^2 \bar{E}}{\partial t^2}$$

Consider a steady-state, sinusoidal electric field bE, propagating within a medium of finite conductivity. The travelling wave must inevitably experience attenuation due to heating, so let us examine a possible solution to the wave equation, that has the form,

$$E = E_o e^{-\alpha z} \sin(\omega t - \beta z)$$

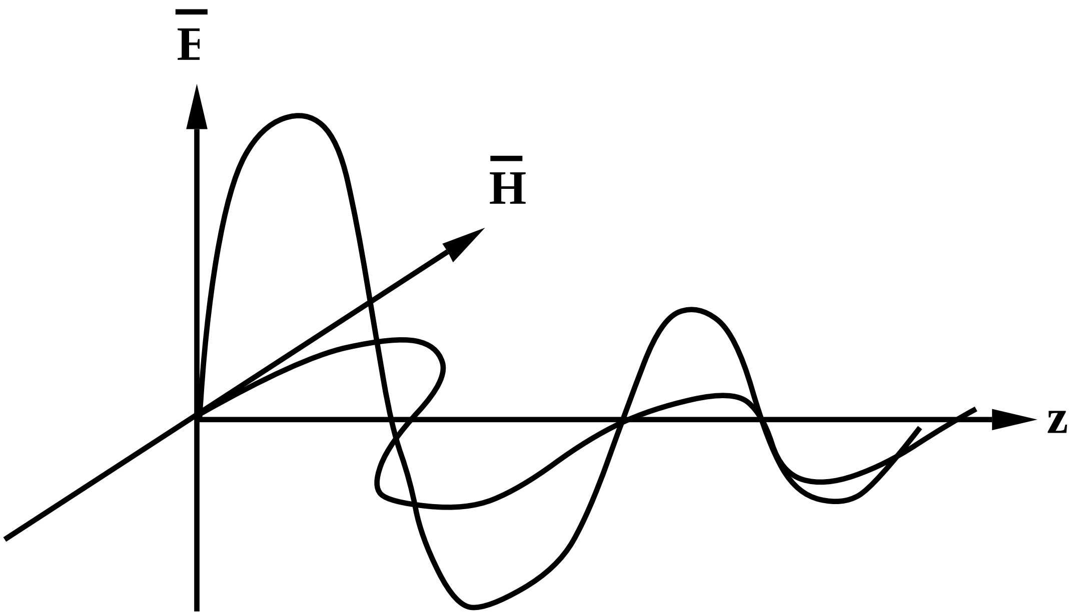


Figure 1 Electric and magnetic fields, mutually at right angles and propagating in z direction.

The field E is shown here to propagate in a direction z, where the direction of E is at right-angles to z, as shown in Figure 1. The attenuation of the wave with distance is chosen to be exponential, $e^{-\alpha z}$, where α is defined as the attenuation constant, while the distance travelled by the wave is determined by β, the phase constant,

$$\beta = \frac{2\pi}{\lambda}$$

and λ (metre) is the wavelength of the propagating field. The frequency at which the wave oscillates is defined by ω, ($2\pi f$) rad/s. Thus, for constant z, E varies sinusoidally, while for constant t, E varies as an exponentially decaying sinewave. An exponential decay is a logical choice, as for each unit distance the wave propagates, it is attenuated by the same amount.

To check the validity of our chosen solution, the function for E must satisfy the wave equation. This validation usefully enables the constants α and β (see expression for E) to be expressed as σ, ε, μ and ω. However, the substitution although straightforward is somewhat tedious, so I will state the commencement and show the conclusion:

Substitute, $E = E_0 e^{-\alpha z} \sin(\omega t - \beta z)$ into,

$$\frac{\partial^2 E}{\partial z^2} = \mu\sigma \frac{\partial E}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

Yes, the function for E is a solution, providing that

$$\beta^2 - \alpha^2 = \mu\varepsilon\omega^2$$

$$\alpha\beta = \frac{\omega\mu\sigma}{2}$$

Solving for α and β,

$$\alpha^2 = \frac{\mu\varepsilon\omega^2}{2}\left[\left(1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2\right)^{0.5} - 1\right]$$

$$\beta = \frac{\omega\mu\sigma}{2\alpha}.$$

Sometimes α and β are written in terms of the propagation constant γ, where $\gamma = \alpha + j\beta$. Thus the constants α, β that govern the propagating field are expressed as a function of the supporting medium, where the parameters (μ, ε, α) are readily available for many materials.

OK, so you may not have followed the detail of the mathematics, but do not worry. It is really only important here to follow the philosophy of the development, that is,

(a) Commence with the established Maxwellian equation set, from which the generalised wave equation for propagation in a lossy material is derived.

(b) Guess at a logical solution for a sinusoidal plane wave, knowing the Fourier analysis allows generalisation to more complex waveforms (at least for a linear medium).

(c) Show that the chosen solution satisfies the wave equation, where the constants α and β follow as functions of μ, ε, σ and ω.

(d) The velocity of propagation v metre/second also follows from ω and β, where the velocity is the "number of wavelengths" travelled in one second, thus

$$v \quad = \quad f\lambda \quad = \quad 2\pi f\left(\frac{\lambda}{2\pi}\right) \quad = \quad \frac{\omega}{\beta}$$

At this juncture, it is now possible to classify materials into good conductors (metals) and poor conductors (lossy dielectrics), though it is important to note that this demarcation is frequency dependent.

(i) **Poor conductor:** (*i.e.*. dielectric materials with very low conductivity) σ is small such that, σ << εω, whereby the expression for the attenuation constant shows α o 0, and the wave experiences minimal attenuation. This condition applies to propagation in both free space and low-loss dielectrics, where the velocity of propagation can be shown to be,

$$v \quad = \quad \frac{1}{\sqrt{(\mu\varepsilon)}} \quad = \quad \frac{300 \,.\, 10^{8}}{\sqrt{(\mu_r\ \varepsilon_r)}} \quad \text{m/s}$$

and μ_r and ε_r, are the relative permeability and permittivity of the supporting medium (μ_r= 1, ε_r = 1 for free space), that is for free space, v = c, the velocity of light. This is the "fast bit" and results in the comment that for audio interconnects, the velocity of propagation *within the dielectric* is so high that signals respond virtually instantaneously across the length of the cable. OK, we will not argue, will we John? John Atkinson smiled, his Linn bounced happily with the platter remaining horizontal.

(ii) **Good conductors:** (*e.g.* copper). Here we assume, σ >> εω *i.e.* ω << σ/ε which for copper implies f < 1.04 . 10^{18} Hz.

σ	=	5.8 . 10^{7} (ohm-m)$^{-1}$
ε	=	8.855 . 10^{-12} farad/m
μ	=	4π . 10^{-7} henry/m

at audio frequencies, copper is a good conductor, where the expressions for α and β approximate to,

$$\alpha \;=\; \beta \;=\; [\mu\omega\sigma/2]^{0.5}$$

values for α and β are identical for a good conductor

and the velocity, $v = \frac{\omega}{\beta}$, whereby

$$v = \sqrt{\frac{2\omega}{\mu\sigma}}$$

very much less than for a material with low conductivity

For copper α, β and v are given by,

$$\alpha = \beta = 15.13\sqrt{f} \quad \text{and} \quad v = 0.415\sqrt{f}$$

Note the frequency dependence of α, β, v !*All significant at audio* !

(*i.e.* at 1 kHz the velocity is 1/25 of the velocity of sound in air . . .)

Skin depth

A parameter often quoted when discussing propagation within a conducting medium is skin depth, δ (metre). δ is defined as the distance an electromagnetic wave propagates for its amplitude to be attenuated by a factor e^{-1}, *i.e.* 8.69 dB (where e is the same e as in an exponential, e = 2.718282, therefore $e^{-1} = 0.3679$).

Recall, $E = E_0 e^{-\alpha z} \sin(\omega t - \beta z)$, thus, for z = δ, then $e^{-\alpha\delta} = e^{-1}$

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\mu\omega\sigma}}$$

i.e. the skin depth δ is simply the reciprocal of the attenuation constant α. It is strictly a convenient definition (see later: "Digression"). but note, the field still exists for z > δ, even though it is attenuated *e.g*. for z = 3.5 δ, just over a 30 dB attenuation is attained.

For copper, it follows, $\delta = (15.13\sqrt{f})^{-1}$. It is also interesting to note that the phase of E, (βz) has changed by 1 radian at z = δ, a far from negligible figure. The following table gives example calculations of skin depth and velocity against frequency.

Table of δ and v for copper against frequency f

frequency f hertz	skin depth δ, mm	velocity v m/s

50	9.35	2.93
100	6.61	4.15
1,000	2.09	13.12
10,000	0.66	41.50
20,000	0.47	58.69

Note the low value of velocity, which is directly attributable to the high value of conductivity of copper, $\sigma = 5.8.10^{7}$ (ohm-m)$^{-1}$

Note also, for information:

σ (silver) = $6.14 \,.\, 10^{7}$ (ohm-m)$^{-1}$

σ (aluminium) = $3.54 \,.\, 10^{7}$ (ohm-m)$^{-1}$

These results suggest a copper wire of 0.5 o 1 mm diameter is optimum (see also later "Digression"). However, the story is far from complete: an electric field travelling within copper has a low velocity and experiences high attenuation, that results in skin depths significant to audio interconnect design.

The frequency dependence of δ (also α and β) should not be underestimated; the copper acts as a spatial filter, the field patterns within the conductor, for a broad-band signal, exhibit a complex form (see Figure 3, for example). Now introduce either/both a spatially distributed non-linearity or discontinuous conductivity, as previously discussed in *HFN/RR* (4), and the defects of cables become more plausible. The distortion residues (linear and/or non-linear) would exhibit a complex, frequency interleaved structure, that could well play to an area of our ear/brain detection process, especially when monitoring an optimally projected stereophonic field. After all, the ear is both non-linear and a Fourier analyser; it would seem strange if we had not evolved a matched, intelligent detector to exploit the complex, possible non-linear, time smeared patterns that must inevitably result. I believe Gerzon, Fellgett and Craven have researched the application of bi-spectral processes as an augmentation to Ambisonics. Is it here that the final, almost hidden, link in our fundamental understanding of audio systems is to be found?

OK, so those who become bored with my earlier analysis may begin again with a new aroused interest. The rest of us will have a Gin and Martini, on *crushed* rocks (rocket fuel), while you complete your revision. Shaken not stirred, please, Ivor.

Let us proceed with the model development. Electromagnetic theory shows a cable to be a wave guide, the conductors acting as "guiding rails" for the electromagnetic energy that propagates principally through the space *between* the conductors, where the currents in the wires are directly a result of the field boundary conditions at the dielectric/wire interface. This may prove a difficult conceptual step for those more accustomed to lumped circuits and the retrogressive 'water pipe' models. However, a wave guide model is correct, irrespective of cable geometry, only the field patterns vary depending upon the conductor shape and their spatial relationship. This theory is not new, it has been widely accepted and practised by engineers for many years.

A propagating electromagnetic wave consists of an oscillation of energy back and forth between the magnetic and electric fields, the energy in the electric and magnetic field must therefore be equal. Think of space (both in general and within the dielectric of the cable) as a distributed LC (oscillator) network.

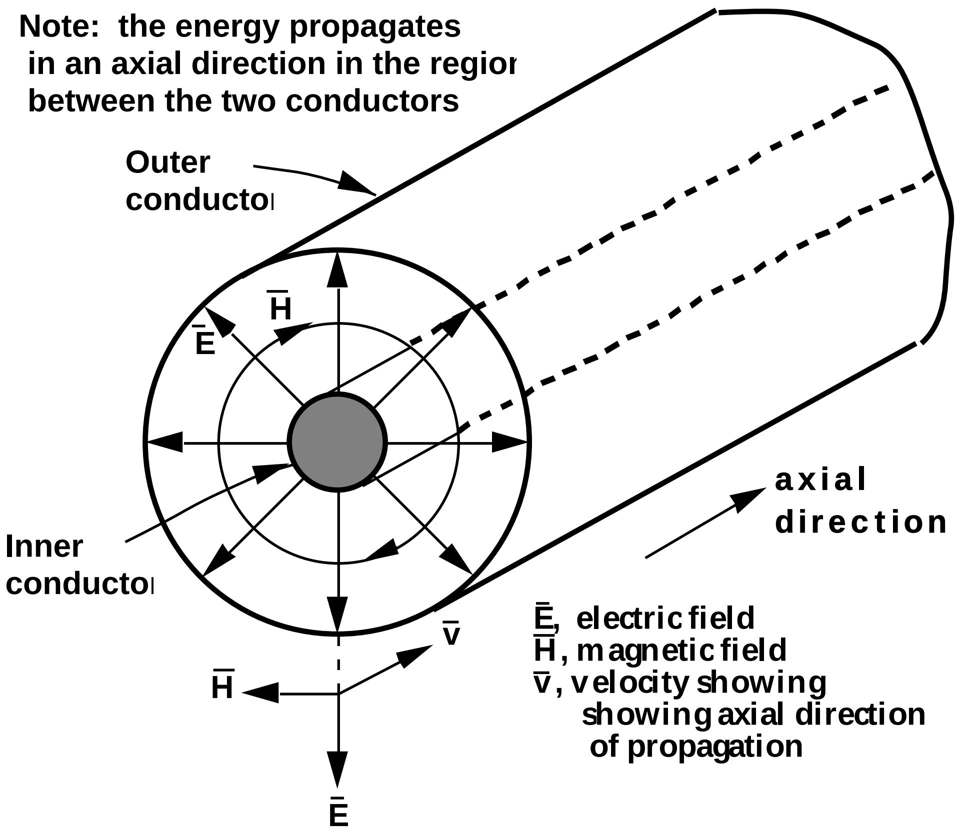


Fig. 2 Cross section of coaxial cable showing radial E fie and circumferential H field.

For example, examine the coaxial cable shown in Figure 2. The electric field is everywhere radial, while the magnetic field forms concentric circles around the inner conductor (Ampere's circuital law). It is important to note that the bE and bH fields are both spatially at right angles to each other and to the direction of propagation, which is along the axis of the cable. This is a direct result of Maxwell's equations.

In an electromagnetic system, the power flow is represented as a density function bP (watt/metre^2), called the Poynting vector, where

bP = bE $_x$ bH

For the coaxial cable, bP is directed axially. Integrate bP over a cross section of area and the *total power* carried by the cable results. The expression for bP can be compared with power calculations in lumped systems, where P = VI (*i.e.* V o bE field, I o bH field).

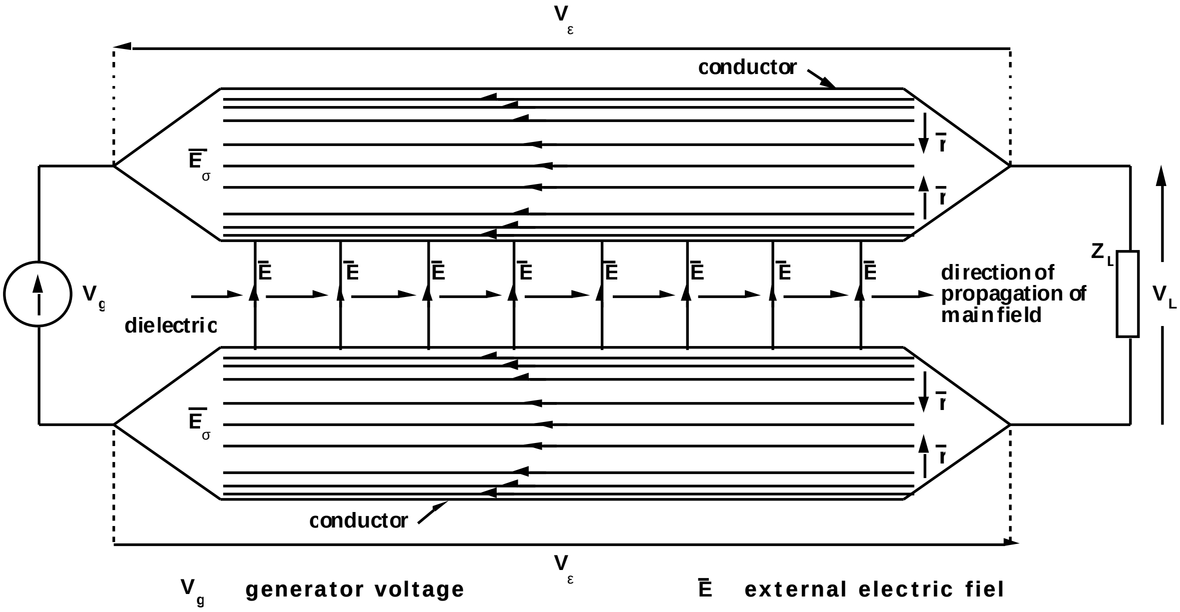
If we assume the two conductors of the coaxial cable are initially ideal, where σ o ∞, then all the electromagnetic energy flows in the dielectric. The bE field does not penetrate the conductors, the skin depth is zero (check with expression for δ) and the conductors act as perfect reflectors (that's why mirrors are coated with good conductors). In this case, there is only a *surface current* on each conductor to match the boundary condition for the tangential, magnetic field bH, at the dielectric/conductor interface(2).

OK, so in your mind you should now visualise a radio wave travelling within the dielectric, being guided by the conductors, where the electric and magnetic fields are both at right angles to each other and to the direction of propagation along the axis of the cable.

However, this example is unrealistic as practical cables have conductors of finite conductivity, σ. Experience shows that such conductors exhibit signal loss, where at a molecular level, friction-like forces convert electrical energy into heat.

As the wave front progresses through the dielectric, the boundary condition is such that the electric field, bE, is not quite at 90° to the conductor surface, which is a direct consequence of the finite conductivity. The wave, in a way, no longer takes the shortest path along the dielectric of the cable and appears to travel more slowly. However, at each dielectric-conductor interface, a refracted field now results within the conductor which proceeds to propagate virtually at right angles to the axis of the cable, into the interior of the conductor. This is the loss field. In other works, the majority of the electromagnetic energy propagates in a near axial direction, within the dielectric, but a much reduced loss field propagates almost radially into each conductor, with the electric field E_σ oriented axially along the length of the conductor. It is this component that is controlled by the internal parameters (μ, σ, ε) of the copper and is ultimately attenuated by conversion to heat. It is here that the story becomes more relevant to audio.

A conductor of finite conductivity causes electromagnetic energy to spill out from the dielectric into the conductor. We should also note that although the main component of energy propagates rapidly within the dielectric along the axis of the cable, the energy spilling out into the conductor propagates much more slowly (see earlier table) and the parameters α, β, that govern the loss wave are frequency dependent, a significant complication. It is the loss wave within the conductor that results directly in current within the copper. We would, therefore, expect a complex current distribution throughout the volume of the conductor, and that is precisely what we get, see Figure 3.



V_g	**generator voltage**	$\bar{E}$	**external electric fiel**
V_ε	**error voltage across conductor length**	$\bar{r}$	**radial direction of propagation of loss field**
$\bar{E}_\sigma$	**loss field in conductor**	V_L	**load voltage**

Fig. 3 Basic field relatioships and direction of propagation of main external field and internal loss field.

Meanwhile, back at the Maxwellian equation set,

$$bJ_\sigma = \sigma bE_\sigma \quad \text{(this is Ohm's law)}$$

That is, a conduction current density bJ_σ is induced axially within the conductors due to the internal electric field, bE_σ, of the loss wave. This axial current is the current we normally associate with cables: the model is compatible with more usual observations of cable behaviour.

Since the electromagnetic energy of the loss wave propagates principally in a radial direction, entering the conductor over its surface area, the current density (which is proportional to bE_σ) is greatest at the surface and decays as the field propagates into the conductor interior. It is this reason why a conductor experiences a *skin effect*, rather than the converse with the current concentrated near the centre of the conductor.

One of the more instructive parameters is the time T_δ, for the sinusoidal loss field, E_σ, to traverse a distance δ within a good conductor, where since,

$$v = \frac{\omega}{\beta} = \frac{\omega}{\alpha} = \omega\delta$$

then

$$T_\delta = \frac{\delta}{v} = \frac{1}{\omega}\bigg|_{\sigma \gg \omega\varepsilon}$$

For example, consider a copper bar where the diameter is greater than the skin depth,

δ	=	0.66 mm	at	10 kHz	:	T_δ	=	15.9 µs
δ	=	2.09 mm	at	1 kHz	:	T_δ	=	0.159 ms
δ	=	6.61 mm	at	100 Hz	:	T_δ	=	1.59 ms

i.e., the lower the frequency and the larger the conductor diameter, the longer T_δ. There is energy storage, it is a memory mechanism.

Observe the importance of discussing principally a time domain model. Our thesis is attempting to demonstrate that a (copper) conductor exhibits significant memory, that influences transient behaviour by time smearing by a significant amount a small fraction of the applied signal.

Consider the cable construction shown in Figure 3. Allow the generator to input a sinewave for a time >> T_δ, to enable the steady-state to be established. The bE field between the conductors responds rapidly to the applied signal, as the velocity in the dielectric between the conductors is high. We are assuming here a terminating load to the cable, so there is a net energy flow through the dielectric. Remember as the wave front progresses, so a *radial loss wave* propagates into each conductor, where the bE_σ field is aligned in an *axial* direction.

Now allow the applied signal to be suddenly switched off. The field between the conductors collapses rapidly, thus cutting off the signal energy being fed radially into the conductors. However, the low velocity and high attenuation of the loss wave represents a loss-energy reservoir, where the time for the wave to decay to insignificance as it propagates into the interior of the conductor, is non-trivial, by audio dimensions.

The bE_σ field within the conductor can be visualised as many "threads" of bE field as shown in Figure 3. The voltage appearing across the ends of each thread, Δe, is calculated by multiplying the bE_σ field by the cable length, L, though more strictly, this is an integral, where

$$\Delta e = \int_{l=0}^{L} E \,.\, dl$$

However, the macroscopic voltage across a conductor, V_ε (*i.e.* that measured externally) is the sum of all these many elemental voltages. Because the field propagates slowly, this summation is actually an average taken over a *time window*, extending over a short history of the loss field. Consequently, when the generator stops, the error signal across each conductor does not collapse instantaneously, the conductor momentarily becomes the generator and a small time-smeared transient residual results as the locally stored energy within each conductor dissipates to insignificance.

Assuming the two conductors are symmetrical, then the total error voltage is $2V_\varepsilon$, whereby the load voltage V_L is related to the generator voltage V_g, by $V_L = V_g - 2V_\varepsilon$. Clearly, $V_\varepsilon << V_g$, however, V_ε takes on a complex and time-smeared form that in practice is both a function of the conductor geometry, cable characteristic impedance, generator source impedance and load impedance, as all these factors govern the propagation of both the main electric field, bE and the electric loss field bE_σ. In practice, unless the cable is terminated in its characteristic impedance, the main field bE will traverse the length of the cable, rapidly back and forth, many times, before establishing a pseudo-steady state. Of course, an optimal load termination unfortunately implies a significant loss field in the conductors.

This argument would suggest that for non-power carrying interconnects, it is better to terminate the generator end of the cable in the characteristic impedance, leaving the load high impedance. The bE field is then rapidly established in the dielectric, without either multiple reflection along the cable length, or a finite power flow to the load spilling out a loss wave into the conductors.

Oh! I see these last comments have raised a question from the floor, from the dark haired Moroccan lady almost wearing a 'belly dancer's costume in the front row: she want to know what happens to the electromagnetic field propagating through the copper conductor when it encounters an abrupt discontinuity in conductivity, and if this has a correlation with defects in copper, attributable to crystal boundary interfaces. (No, Ken, this is not the appropriate time to recommend the use of Gold Lion KT77s.)

Consider for a moment a long transmission line terminated in its characteristic impedance. Electromagnetic energy entering the line will then propagate in a uniform manner, finally being totally absorbed in the load (just as a VHF aerial cable which is terminated in 75 Ω). If, however, the termination is in error, then a proportion of the incident energy will be reflected back along the cable towards the source. In extreme cases, where there is either an open or a short-circuit load, then all the incident energy is reflected, although with a short circuit the sign of the bE field is reversed on reflection, thus cancelling the electric field in the cable and telling the source there is a short circuit termination. The point to observe, is that a discontinuity in the characteristic impedance results in at least partial reflection at the discontinuity, which will distort the time-domain waveform. This reflective property of a change in characteristic impedance can be used, for example, to locate faults in long lengths of cable, by using time domain reflectometry, that is, a pulse is transmitted along the cable and the returned partial echoes from each discontinuity are measured, their return times then locate the fault. It is the same principle as radar, though the universe is a narrow cable.

Similarly, for a wave travelling in copper, a discontinuity in impedance leads to partial reflection centred on the discontinuity. This effect must therefore be compounded with an already dispersive propagation, *i.e.* different frequencies propagate at different velocities thus time smearing the error signal or loss wave in the conductor. OK, let's now play to the gallery . . .

This observation certainly gives some insight into the effects of crystal boundaries within copper, where each boundary can be viewed as a discontinuity in σ and corresponds to zones of partial reflection for the radial loss field. Note however, that this property is *not necessarily non-linear*. We do not have to invoke a semiconductor type non-linearity to identify a problem, we are talking probably of mainly linear errors. So we would not necessarily expect a significant reading on the distortion factor meter or modulation noise side-bands on high-resolution spectral analysis, for a stead-state excitation. However, just as with loudspeaker measurements, amplitude-only response measurements do not give a complete representation of stored energy and time delay phenomena. We would require very careful measurements directly of the errors with both amplitude and phase, or of impulse responses in the time domain. Following the comments on the error function in an earlier Essex Echo(5), direct measurements of the output signal will yield, in general, insufficient accuracy to allow a true estimate of the system error. This point is worth thinking about, re-read my earlier comments in the first Echo (5). Ideally, we need to assess the actual current distribution in the conductors, or at least to measure the conductor error directly.

A smile now appeared on the young lady's face, it was Alice through the Looking Glass all over again - she now understood the subtle distortion in John Atkinson's reflection. As she relaxed, her large crystal diamond of high permittivity fell to the floor.

The final stage in the development of our model, is to account for copper conductors of finite thickness, where the thickness may well be much less than the skin depth. Just as a wave travelling in air when confronted by a short-circuit is reflected, so a wave travelling in a conductor, that encounters an open circuit (*e.g*. copper-air boundary) also undergoes reflection and therefore passes back into the conductor, undergoing further attenuation. However, the boundary condition requires a reversal of the magnetic field, thus providing the thickness of the conductor is much less than the skin depth, the incident and reflected bH fields nearly cancel and the conductor exhibits a lower internal magnetic field. Consequently, there is predominantly an axial electric field and corresponding conduction current, the conductor behaves nearly as a pure resistor, *i.e.* the magnetic field hence, effectively, inductive component, is reduced to the pseudo-static case. The current distribution is nearly uniform. The conductor has lost its memory.

Digression
For completeness, let us now take a more conventional look at skin depth, to demonstrate that our model is consistent (OK John, not running out of time yet?)

A direct effect of skin depth is the well-known phenomena that the current in a conductor resides near the surface at high frequency, where this notion is perfectly consistent with our model.

A reason for specifying δ as the distance travelled whereby the field has decayed a fraction e^{-1} is as follows:

Let the current density be given by: $J_\sigma = J_o e^{-\alpha z} \sin(\omega t - \beta z)$

The total instantaneous current, I, in a strip of conductor of width ΔY but of infinite extent in z is then,

$$I = \Delta Y \int_{z=0}^{\infty} J_0 e^{-a\, z} \sin(v\, t - \text{ß}z)\, dz$$

Evaluating the integral, putting $\alpha = \beta = \delta^{-1}$ gives,

$$I = d\, \Delta Y\, J_0 \sin(v\, t - \pi/4)$$

The result above shows that the amplitude of the total conduction current is {δ ΔY J_o}, it is as if a uniform current density existed only for z = 0 to δ, but was everywhere else zero for z > δ. This leads to our colloquial notion of skin depth, but observe how the -π/4 phase shift (-45 degree) with respect to the surface-current density, disguises the propagation of the conduction current that is internal to the conductors. So we see there is a logical foundation for our definition of skin depth.

This convenient but approximate view-point of the current distribution being concentrated in the skin depth allows us to estimate an approximate impedance for the conductor, based upon the principle that the conductor is now only of thickness δ. Note however, that this approximation completely removes the more subtle structure of our model.

Imagine a cylindrical conductor of diameter D metre and length L metre where δ < 0.5 D. Picture the skin depth as an annulus as shown in Figure 4. We may write the modulus of the dc and ac impedances $|Z_{dc}|$, $|Z_{ac}|$ measured across a length of L of this conductor, as

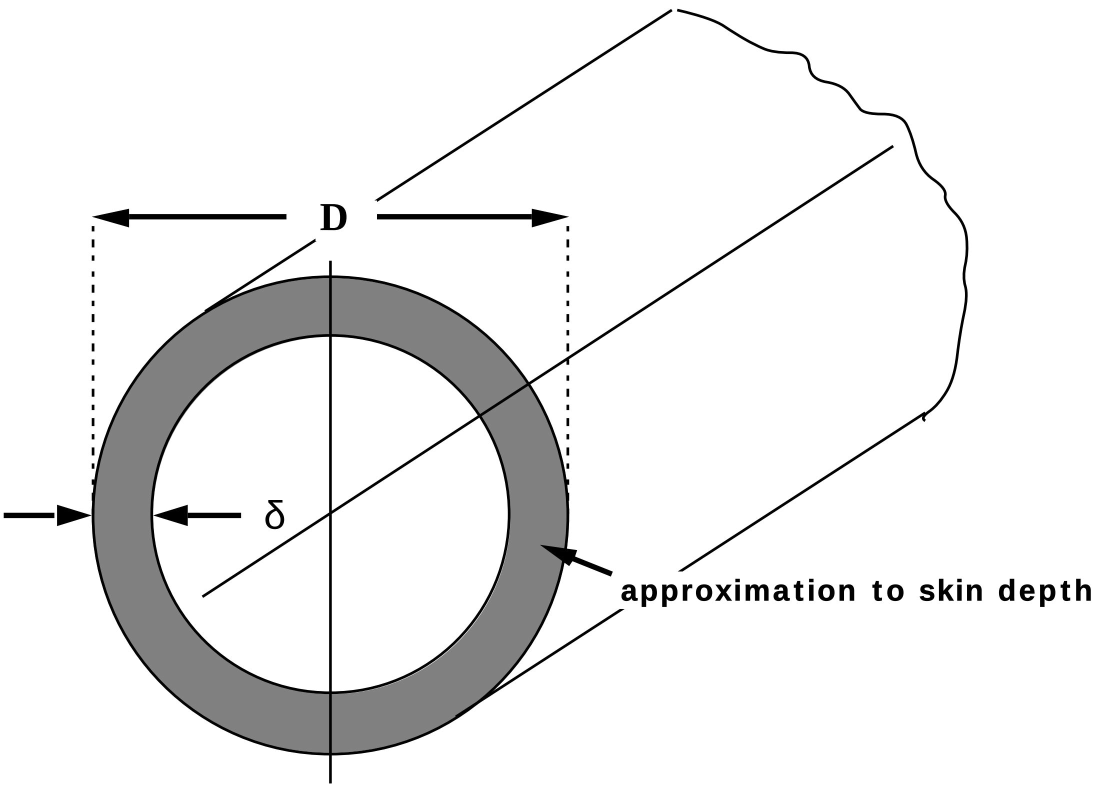


Figure 4 Cylindrical conductor showing approximation to skin depth.

$$|Z_{dc}| = \frac{4\,L}{s\,\pi D^2} \quad \text{and,} \quad |Z_{ac}| = \frac{L}{s\,\pi d\,(D - d)}$$

but substituting for δ from our earlier result.

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\mu\omega\sigma}}$$

then for the case where δ << 0.5 D,

$$\frac{|Z_{ac}|}{|Z_{dc}|} = \frac{D^2}{4\,d\,(D - d)} \approx \frac{D}{4\,d} \approx \frac{D}{4}\sqrt{\frac{mv\ s}{2}}$$

Returning to Maxwell, we could also show that when the impedance is proportional to √(frequency). that there is a 45° phase shift between voltage and current, but when δ approaches D/2 and ultimately, δ > D, then Z_{ac} is substantially resistive with zero phase, which is our usual low frequency model of a piece of wire.

Hence, where δ = D/2, we expect to observe a transitional region in the effective conductor impedance, *i.e.*. $|Z_{ac}| \approx |Z_{dc}|$. This critical frequency f_c follows approximately from our expression for α by putting α = 1/δ = 2/D, whereby

$$f_c = \frac{4}{\pi\mu\sigma D^2}$$

e.g. for a diameter of 0.8 mm f_c = 27 kHz. However, this is only approximate as the cylindrical geometry has not been fully accounted for in the analysis. Hence, thin conductors behave more like resistors over the audio band, whereas thick wires have a complex impedance, rather like the "square root of an inductor" *i.e.* the impedance modulus is proportional to √f, the phase – 45°.

In this latter discussion, we have interpreted our model in the steady-state, and as a lumped impedance. However, we should not lose sight of the time-domain model and the generalisation to a discontinuous or granular conductivity. Again, as observed in the earlier Echo(5), steady-state analysis though correct, can limit our appreciation of a system. We would not expect to observe anomalies on steady-state tests easily as in the main they are hidden from view, the test is insensitive. The observations should be made when the signal stops, at the end of a tone burst for example, and the error signals displayed by using decay spectra, following loudspeaker measurement practice.

In developing our model, we have concentrated on the loss mechanisms inherent in the conductors. We have not discussed the characteristic impedance observed at the input of the interconnect. This is a direct result of the amount of energy in the electric and magnetic field needed to "fill" the cable *i.e.* the propagating energy within the cable system. Ultimately, the energy loss in the interconnect is a function of the characteristic impedance and the length and load termination as these directly influence the loss field, hence conductor current, hence voltage across the cable length. The load impedance that terminates the line is mapped into the interconnect error mechanisms which is particularly relevant with loudspeaker loads.

The detailed characteristics of the dielectric material are also important as the model shows that the dielectric supports the majority of the signal during its transportation across the cable (which can take many passes if the cable is not optimally terminated). Dielectric-loss has been cited as a contributory factor, which can be modelled as an equivalent frequency dependent, but low conductivity σ_d where $\sigma_d = \omega\varepsilon$ (Power factor), and power factors vary(2) from typically ~0.0005 to

0.05. The attenuation and phase constants then follow as $\alpha_d = 0.05\omega\sqrt{(\mu\varepsilon)}$ (Power factor), $\beta_d = \sqrt{(\mu\varepsilon)}$. However, it is difficult to see how these results affect audio cables from this simplistic appraisal. A more detailed study of the permittivity of dielectrics is required. Directional wave characteristics could well affect the loss wave launched into the conductors. But times is running out . . .

Conclusions

The basic elements of our model are now complete, where we propose the internal loss fields that propagate *within* the conductors are at least partially responsible for some claimed anomalies. The points to emphasis are as follows:

(a) The loss component propagates at *right angles to the axis of the cable i.e*. radially into the conductors.

(b) The loss field gives rise to the corresponding internal current distribution along the axis of the conductor ($bJ_\sigma = \sigma E_\sigma$). Note for the loss component, that although the direction of propagation is radial, the bE_σ field is at right-angles to the direction of propagation of the radial loss wave and is along the conductor axis. This induces an axial conduction current and is the component of current normally experienced.

(c) The velocity of propagation within the conductor (copper) is both very slow and frequency dependent, consequently, different frequencies propagate at different velocities *i.e.* the material is highly dispersive and acts as a spatial filter.

(d) The velocity of the loss field is directly dependent upon the conductivity σ and pemeability μ, which should be noted for magnetic materials. Usual analysis assumes σ to be a smooth and continuous function. However, crystal boundaries suggest discontinuities in σ, such that the conductors appear more like stranded, though disjointed, wire where such discontinuity represents a point of at least partial reflection and field redistribution.

(e) There is a problem even if σ is a linear but discontinuous function. However, non-linearity due to partial semiconductor diode boundaries would lead to a very complex, frequency interleaved intermodulation that could be governed by bi-spectral processes, to which the ear/brain may have a significant sensitivity; such residues would of course be at low level.

(f) Stranded conductors appear to be a poor construction, when viewed by this model. The loss component propagates against the strands and will experience discontinuities of air/copper that are inevitably random. This is comparable to a large-scale granularity, where crystal boundaries represent possibly a similar structure but within the copper. A single strand of large crystal copper will behave more as a simple impedance as outlined in the "Digression". Normal simplified theory and actual conductor performance merge, where at a diameter *circa* 0.8 mm the conductor becomes closer to a low-valued ideal resistor, at audio frequencies.

(g) Irregularities in cable construction and directional wave properties in the dielectric could well lead to differences in the bE_{σ} field patterns, hence current distribution within the conductors, depending upon which end is the source. (I wonder if current vortices can result, like whirlpools in a stream of water?) The exact nature of the loss field (error field) would, in principle, exhibit differences and thus allow the cable to have a directional characteristic in that the error is not mirror symmetric. For example, slight variations in diameter, or indeed internal crystal structure, may well occur in manufacture due to stress fields. Such effects however, would appear to be in the domain of error of errors, and of an extremely subtle nature, where steady-state measurements would exhibit poor measurement sensitivity, yet the residues from impulse testing would contain low energy. In other works, very difficult to measure.

(h) Since all materials within the cable construction, including surface oxidisation of the conductor indirectly affect the boundary conditions, hence loss field, we would expect each element to contribute to performance.

(i) The time taken for the field to propagate to the skin depth δ, is longer at low frequency. Thus, thick conductors would appear more problematic at low frequency, showing a greater tendency for time dispersion (over-hang).

(j) $\delta = [2/(\omega\mu\sigma)]^{0.5}$, magnetic conductors have μ-dependent skin depths and μ is partially non-linear. This needs investigation; it suggests magnetic conductors should be avoided. (Of course, I would never admit to checking passive components with a magnet . . .)

(k) The ear-brain sensitivity to particular complex, high-order frequency-dependent intermodulation distortion requires careful research, using possible interconnect defects as a basis for identifying classes of error and of error correlation mechanisms.

(l) Stepping back and observing the problem macroscopically, it appears cable defects have their greatest effect under transient excitation rather than within the pseudo steady-state of sustained tones. Transient edges are effectively time smeared or broadened albeit by a small amount, where this dispersion is a function of both the signal and the properties and dimensions of the conductors. Amplitude frequency response errors in the steady-state are at a level that is insignificant when listening to steady-state tones. Their significance however, when mapped via the error function onto transient signals may well be of greater concern, particularly when the errors are monitored optimally in stereo. In this sense, we support the Editorial comments recently made by John Atkinson on the importance of maintaining transient integrity at the beginning and end of sequences of sound, rather then worrying about slight relative level errors in the pseudo steady-state of a sustained tone, or a slight change in harmonic balance. It's the old story of measuring a frequency and phase response with insufficient accuracy to extract the true system error and then misinterpreting the significance of that error: check out the error function[5].

(m) At audio frequencies, axial propagation within the dielectric is usually not considered important as interconnects are generally much shorter than a wavelength, even at 20 kHz. However, we have directed our attention to the loss field *within* the conductors, where, due to the slow velocity, cable dimensions comparable to wavelength are significant. It is suggested that this viewpoint is usually not considered, where skin depth is rarely appreciated in audio circles to be a propagation phenomena.

From these observations, we conclude that conductors should be sufficiently thin that only a fraction of a wavelength at the highest audio frequency is trapped within the conductors. The external propagating fields should be distributed as uniformly as possible over the whole surface of the conductor. The composite cable should be tightly wrapped, to prevent external mechanical vibration from modulating the characteristic impedance (shaking wires, coils and interconnects in loudspeaker systems, for example). (Thinks: could the crystal boundaries be vibration dependent? . . . time to stop. Oh, everyone but the Moroccan girl and Ken has left!)

This article has tried to describe a more rigorous model (finely etched with a little speculation) for cable systems by reviewing some fundamental electromagnetic principles. It is important not to make engineering simplifications too prematurely when evolving a model. Clearly, we have made some approximations as field patterns can be highly complicated, depending on cable geometry's and internal material behaviour at a molecular level (and I keep thinking of current vortices). Nevertheless, there is sufficient evidence to suggest a cable's performance is not as simple as it first appears, often because the operation is viewed too approximately and our notions of lumped circuit elements (discrete Rs, Cs, Ls etc) warp our thinking, especially with respect to skin depth. To me, the most striking observation is the slow, frequency dependent velocity of a wave travelling in a conductor; it's rather like launching a sound wave into a room and waiting for the reverberant field to decay. Also, a high conductivity and permeability makes the conductor appear much larger on the inside and crystal boundaries act as partitions within that space. TARDIS o Transient And Resistance DIStortion. Now, who said that? Famco of France have just send me some Vecteur cable(6), conductor diameter 0.8 mm, large crystal copper, immaculate screening, little arrows . . . Now Ken, what was that about KT77s? So you've heard that all electromagnetic waves are discrete packages of energy and mercury has a non-crystal structure. OK, OK . . . I'll turn up the volume and use only mercury capillary interconnects.

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